Optimal mixed taxation, credit constraints and the timing of income tax reporting

Robin Boadway\textsuperscript{1}, Jean-Denis Garon\textsuperscript{2}, and Louis Perrault\textsuperscript{3}

\textsuperscript{1}Queen’s University
\textsuperscript{2}Université du Québec à Montréal
\textsuperscript{3}Georgia State University

January 12, 2018

Abstract

We study optimal income and commodity tax policy with credit-constrained low-income households. Workers are assumed to receive an even flow of income during the tax year, but make tax payments or receive transfers at the end of the year. They use their disposable income to purchase multiple commodities over the year. We show that differentiated subsidies on commodities can be welfare-improving even if the Atkinson-Stiglitz Theorem conditions apply. The extent of the differentiation depends on the cost

\textsuperscript{*}We thank Katherine Cuff, Philippe De Donder and Christian Moser for their comments on an earlier draft. We also thank participants at the 2017 IIPF Congress (Tokyo) and the 2017 National Tax Association Conference (Philadelphia).
of transferring resources between periods and the ratio of costs coming from income
effects of the subsidies through different levels of consumption made by constrained
individuals of each commodity.
1 Introduction

Many government transfer programs are income-tested and delivered through the tax system. Examples include refundable tax credits that decline in income, such as the Earned Income Tax Credit, the Additional Child Tax Credit and the Health Coverage Tax Credit in the U.S.; the Working Income Tax Benefit, the Canada Child Benefit and the Goods and Services Tax Credit in Canada; and the Working Tax Credit, the Child Tax Credit, and the Universal Credit in the U.K. A key feature of these transfer programs is that entitlements cannot be fully determined until the taxpayer’s income tax form has been filed and approved by the tax authorities. In the above examples, transfer payments are paid periodically in a given year based on taxable income (or family income) of the previous year. In some cases, adjustments can occur while the transfers are being received if the taxpayer’s circumstances change in a way that can be verified by the government, such as childbirth or change in employment or disability status.

The consequence is that transfer recipients’ income flow is lumpy. Those with low enough income to be eligible for a transfer from the government will have low—possibly zero—income during the year and a large transfer starting after the year ends. Individuals who anticipates a transfer would like to smooth their consumption stream over the year by borrowing. However, they may be precluded from doing so by a credit constraint. Financial institutions may be unwilling to lend to them except at exorbitant interest rates, especially if they do not have a credit rating or if the financial institution cannot verify the expected transfer.

We adopt an optimal income and commodity tax perspective to study policy responses to this issue. The informational assumptions of optimal taxation accord well with the problem. The model we use is stylized and meant to capture the essential features of the information constraint faced by the government and the credit constraint faced by transfer recipients. Unlike in the standard optimal income tax setting, we assume that individuals receive an even flow of income during the tax year, but make tax payments or receive transfers at the
end of the year. Individuals use their disposable income to purchase a flow of multiple commodities over the tax year. The government knows only the workers’ labor incomes at the end of the year. However, following Guesnerie (1995), we assume that the government observes all anonymous transactions on commodity markets and can impose a set of linear commodity taxes or subsidies at the time the purchases occur. Therefore, if the government wants to undertake some redistribution before the end of the fiscal year, implicit transfers can be made through commodity subsidies and could be targeted to the intended individuals by a differential rate structure.

Our main focus is on the case where individuals are credit constrained which can prevent them from smoothing their consumption over the fiscal year. The credit constraint becomes especially relevant when the government’s redistribution scheme implies paying transfers at the end of the year. With perfectly functioning credit markets, those anticipating transfers would borrow throughout the year to smooth the consumption financed by their future transfer. Then, the standard results of optimal tax theory would hold, including the well-known Atkinson & Stiglitz (1976) theorem when labor and consumption are weakly separable. However, when transfer recipients face a binding credit constraint that precludes them from smoothing their consumption, giving transfers at the end of the year does not achieve the government’s redistributive objectives earlier in the year. And, the government cannot provide optimal transfers before the end of the tax year since it does not have the required information to determine who is entitled to them. We show that differentiated subsidies on commodities can be welfare-improving even if the Atkinson-Stiglitz Theorem conditions apply.

The idea that consumption tracks income due to credit constraints is well established. For example, in the buffer-stock model of Deaton (1991), consumers’ inability to borrow and impatience predicts that consumption will track income and that credit constraints can

---

1In practice, tax remittance are often made throughout the year by employers through payroll deductions, but this only applies for taxpayers and not transfer recipients. Ignoring these remittances will have no effect on our analysis since those who pay taxes face no credit constraint.
be binding. Various studies using U.S. data confirm this. Using evidence on caloric intake of food stamp recipients, Shapiro (2005) finds that the short-term discount rate of these individuals is very high and hardly reconciliable with geometric discounting. Studying the effect of stimulus payments from the 2001 tax cut episode to explain the phenomenon of ‘wealthy hand-to-mouth’ who own mostly illiquid assets, Gruber (1997) finds evidence that unemployment insurance, which is paid on a frequent basis, significantly smooths household consumption. Parker (1999) finds that consumers do not perfectly smooth their demand for goods when they expect a change in their income (although, in their case, the complexity in the tax code may be at stake). More recently, Aguila et al. (2017) found in a natural experiment that smoothing cash-transfers over the year facilitates consumption smoothing. In particular, they find that more frequent cash-transfer programs are associated with more consistent spending on basic needs, such as food and doctor appointments.

Another source of evidence comes from household behavior during the months when the Earned Income Tax Credit (EITC) is received. McGranahan & Schanzenbach (2013) find that households who are eligible for the EITC spend relatively more on healthy items during the months when most refunds are paid. Among these healthy items one finds vegetables, meat, poultry and dairy products. In a recent survey paper, Nichols & Rothstein (2016) stress that “[households] are often unable to borrow at reasonable interest rates (as evidenced by the high take-up of extremely high interest refund anticipation loans). If credit constraints are binding, a lump-sum payment has a smaller effect on the household’s utility than would a series of smaller payments throughout the year.” They also note that until 2010, EITC recipients could apply for a partial advance payment throughout the year. Although a small proportion of individuals opted-in, the most plausible explanation for taking up the credit would be that individuals are severely credit constrained.

In a recent work, Baker (2017) finds that the income elasticity of consumption is significantly higher for highly indebted households (after controlling for net assets). He concludes that “credit constraints play a dominant role in driving differential household consumption
responses across households with varying levels of debt.” Also, using data from households who experienced a temporary income reduction during the U.S. federal government shutdown in 2013, Baker & Yannelis (2015) find indications that households who have better access to credit or who have accumulated more savings exhibit significantly smaller spending reductions during the transitional shock.

In the following sections, we study optimal income and commodity tax policy with credit-constrained low-income households in a standard nonlinear income taxation setting. The model features several skill-types of households who supply labor and consume two commodities. To simplify matters, we assume that transactions can occur at two discrete points: in the middle of the period and at the end. Preferences are weakly separable so in the absence of credit constraints, optimal commodity taxes will be uniform at indeterminate rates given that proportional commodity taxation is equivalent to proportional income taxation. The two commodities are not consumed in the same proportions by different skill-types, and this will lead to differential commodity subsidization in the presence of credit constraints. The credit constraint will take the simplest of forms. As well, for reasons to be explained, it will be costly for the government to make budgetary expenditures before the end of the period. Doing so requires it to borrow against its end-of-period tax revenues.

In principle, the government could make a uniform lump-sum payment to all persons at the beginning of the period. Combining a lump-sum transfer with non-differentiated commodity taxes would be equivalent to a linear progressive tax system and would allow the government to redistribute at the beginning of the period even if it had no information on individuals’ incomes. If preferences were weakly separable in goods and labor and quasi-homothetic in good — the Deaton (1979) conditions — non-differentiated commodity taxes would be optimal, and this would have implications for our analysis. In our analysis, we assume that the government does not use a uniform lump-sum transfer at the beginning of the period. In particular, we assume that all components of the direct tax-transfer system are implemented at the end of the period. We return to a discussion of beginning-of-period
2 Model

There are $N$ types of individuals who are indexed by $i \in \{1, \ldots, N\}$. The number of type–$i$ individuals is $n_i$, each of whom has exogenous productivity $w^i$. The whole population is normalized to one so that $\sum_{i=1}^{N} n_i = 1$. The economy lasts for one period, which we can think of as a tax year. We divide the period into two sub-periods $t = 1, 2$, and assume that each individual works with the same intensity in both sub-periods and earns a gross income $Y^i/2$ in each. At the end of $t = 2$, a type–$i$ individual pays an income tax $T^i$ (or receives a transfer if it takes a negative value). When individuals choose their labor supplies ex-ante, they know their end-of period income tax liability and therefore their disposable income over both sub-periods.

We use the methodology of Christiansen (1984) to introduce consumption of commodities into the model. In each sub-period $t$, type–$i$ individuals choose a consumption bundle consisting of two goods $(c^i_t, d^i_t)$. The producer prices of goods $c$ and $d$ are set to unity, and the consumer prices can include a commodity tax, which can equivalently be either per unit or ad valorem: $q_c \equiv 1 + t_c$ and $q_d \equiv 1 + t_d$. Commodity taxes $t_c$ and $t_d$ are the same for both sub-periods and for all individuals since otherwise arbitrage opportunities would exist. An individual’s utility function is assumed for simplicity to take the following additive form:

$$U^i(c^i_t, d^i_t, Y^i) = \sum_t u(c^i_t - \bar{c}, d^i_t) - h\left(\frac{Y^i}{w^i}\right)$$

(1)

where $Y^i/w^i$ is labor supply in each of the two sub-periods, and $h(\cdot)$ is a strictly convex cost or disutility function. The function $u(\cdot, \cdot)$ is the per-period utility of consuming the bundle of goods. To ensure that commodity tax differentiation is not a by-product of nonlinear Engel curves, we sometimes assume that $u(\cdot, \cdot)$ is quasi-homothetic in $c^i_t$ and $d^i_t$ by introducing a
basic need \( \bar{c} \) on good \( c \) and letting \( u(\cdot, \cdot) \) be homothetic in \( c_i^t - \bar{c} \) and \( d_i^t \). The quantity \( \bar{c} \) could stand for a minimal quantity of food or shelter. For simplicity, we assume that individuals do not discount their utility across periods, which does not restrict our results. Note that although individuals supply labor in both sub-periods, the disutility of labor supply is defined over total (annual) labor supply. Since commodities are separable from labor or leisure in the utility function (1), the Atkinson-Stiglitz Theorem would apply in this model in the absence of a credit constraint, as we confirm below.\(^2\)

We introduce imperfections in the credit market in the form of a credit constraint. The credit constraint applying in the first sub-period is

\[
q_{c} c_{i1} + q_{d} d_{i1} \leq \frac{Y_{i2}}{2} + \phi,
\]

where \( \phi \) is exogenously given. In what follows, we assume \( \phi = 0 \) so individuals are precluded from borrowing. Individuals have access to a competitive credit market if they want to save or are able to borrow. Those who save do so at rate \( r \) and those who borrow do so at rate \( \bar{r} \), with \( r \leq \bar{r} \). This reflects the cost of financial intermediation. For an individual \( i \), we denote by \( r_i \in \{r, \bar{r}\} \) depending on whether, in the optimum, he is respectively a net saver or borrower at \( t = 1 \). If the government borrows, it can do so at rate \( r_g \), meaning that it borrows at a higher rate than the risk-free rate at which individuals can invest their short-term savings.\(^3\)

Under these assumptions, we shall see that the two sub-period setting gives the same solution as a standard Mirrlees problem when there is no credit spread, that is, when \( r = \bar{r} = r_g \). This is our benchmark case which we study first. Then, we introduce a borrowing constraint that prevents individuals from using more than \( \phi \) dollars of their end-of-year transfers as a collateral when applying for a loan. As mentioned, a simple case is when

\(^2\)The model assumes that individuals commit to their labor supply and that labor supply is the same across periods. This does not drive the results and simplifies the analysis.

\(^3\)In particular, this prevents the fiscal policy from being a Ponzi scheme and eliminates arbitrage opportunities.
\(\phi = 0\), which mimics the corner solution one would obtain if borrowers faced an interest rate \(\bar{r}\) that is prohibitively high. Given that our model abstracts from solvency issues and financial risks related to lending to individuals, this is a simple way to introduce credit market frictions without explicitly modeling solvency risks.\(^4\)

To be precise, in the case where there is no credit constraint, the annual budget constraint for a type-\(i\) individual is (from an end-of-year standpoint)

\[
(q_c^1 + q_d^1)(1 + r_i) + q_c^2 + q_d^2 \leq \frac{Y^i}{2}(1 + r_i) + \frac{Y^i}{2} - T^i.
\]

Since individuals earn \(Y^i/2\) every sub-period and only pay their taxes (get their transfers) \(T^i\) at the end of the year and they can make transactions in the financial markets, the nonlinear tax problem amounts to choosing annual disposable income defined as

\[
I^i \equiv \left(\frac{2 + r_i}{2}\right)Y^i - T^i.
\]

Therefore, one can rewrite the individual’s annual budget constraint as

\[
(q_c^1 + q_d^1)(1 + r_i) + q_c^2 + q_d^2 \leq I^i.
\]

\[2.1\] Tax normalizations

In the standard static optimal income and commodity tax analysis, uniform commodity taxes are equivalent to a proportional income tax. This implies that the absolute level of commodity taxes is indeterminate: reducing commodity tax rates proportionately and increasing the income tax rate by the same amount will have no effect on equilibrium outcomes. Commodity taxes can then be normalized by, for example, setting one commodity tax rate

\(^4\)A more complex model would involve risk. Then, it would be costlier to banks to lend to individuals and the interest rate for borrowers would be high. This would give us the same intuition, but would significantly complicate the problem.
to zero. In our setting, this is not possible if credit constraints are binding. That is because while commodity taxes are paid on purchases in both sub-periods, income taxes apply only at the end of the periods.

To illustrate, suppose the government imposes undifferentiated commodity taxes $t_c = t_d$. In the absence of binding credit constraints and assuming no interest rate spread between borrowing and lending, it can reach the same allocation by taxing everyone’s yearly income at the proportional rate $t_Y = t_c/(1 + t_c) = t_d/(1 + t_d)$. In this case, we can normalize one consumption tax to zero and let the flat revenue-collection component be captured by the proportional tax on income (leisure). Recall, however, that income taxes are collected at the end of the period, while commodity taxes apply in each sub-period. Thus, the time stream of tax liabilities will differ under the two systems. A uniform commodity tax system will generate tax liabilities in both sub-periods while income tax revenues will be paid at the end of the period. This difference in timing has no real effect in the absence of credit constraints and interest rate spreads. The analogous result applies in the case where a uniform commodity subsidy is applied.

However, when an individual’s borrowing constraint binds, this equivalence does not hold. The income tax is not paid in the first period, so with $\phi = 0$ the binding credit constraint (2) becomes

$$
(1 + t_c)c^i_1 + (1 + t_d)d^i_1 = \frac{Y^i}{2}.
$$

A proportional increase in commodity tax rates will tighten the credit constraint in (5), while a corresponding proportional decrease in the income tax rate will not undo this tightening. Therefore, proportional commodity taxes or subsidies are not equivalent to proportional income taxes or subsidies. The absolute level of commodity tax rates matters so we cannot normalize one rate to zero.

Note further that (5) does not contain a tax on its right-hand side. Therefore, if the government want to tax income specifically in the first period, it has to do it through the
taxation of goods. Similarly, if he wants to redistribute in the first period, it has to do it either through a subsidy on goods or through a uniform lump-sum subsidy to all individual in the first sub-period (since it cannot identify individuals by type then).

In what follows, we treat the absolute levels of commodity tax rates as government policy variables along with the nonlinear income tax system. Unlike in the standard models of optimal income and commodity taxation, our analysis yields a well-defined tax mix.

2.2 Government’s budget constraint

The government’s budget constraint in absolute terms in end-of-period values is

\[ \sum_i \left( \left( \frac{2 + r_i}{2} \right) Y^i - I^i \right) + (1 + r_g)(q_c - 1) \sum_i c_1^i + (q_c - 1) \sum_i c_2^i + (1 + r_g)(q_d - 1) \sum_i d_1^i + (q_d - 1) \sum_i d_2^i = R. \] (6)

where \( R \) is an exogenous revenue requirement. Note that the discount factor \( r_g \) is used to obtain the end-of-period values of commodity tax receipts in the first sub-period. That is because we are assuming that the government is a net borrower. If it subsidizes commodities in the first sub-period, it must borrow at the rate \( r_g \) to finance those subsidies. Some of the benefit of the subsidies accrues to high-income individuals who are savers and obtain a return \( r \) on their savings. The fact that \( r_g > r \) makes it socially costly to transfer resources to them in \( t = 1 \). By the same token, if the government taxes commodities in the first sub-period, it reduces its borrowing and the saving of high-income individuals also decreases, which again saves resources since \( r_g > r \). However, the credit constraint is tightened for low-income individuals for whom it binds.
3 Optimal tax mix

We derive the government’s optimal tax structure using a standard mechanism design problem for income taxes augmented by a choice of commodity tax rates. The government offers bundles of income and disposable income \((Y^i, I^i)\) intended for types \(i\), where income is earned equally over the two sub-periods. Then, using (3) taxes paid at the end of the period are residually given by \(T^i = (2 + r_i)Y^i/2 - I^i\), where \(T^i\) can be negative for low-productivity types. The government also chooses \(t_c\) and \(t_d\), or equivalently \(q_c\) and \(q_d\). As we shall see, when an individual is credit constrained in the optimum, the optimal price ratio \(q_c/q_d\) will generally differ from unity. We begin by characterizing individual behavior, and then turn to the government’s problem.

3.1 Individual behavior

We solve the type-\(i\) individual’s problem in two steps in reverse order. In the second step, knowing \(Y^i, I^i, q_c, q_d\), the individual chooses bundles \((c^i_t, d^i_t)\) for \(t = 1, 2\). In the first step and anticipating the outcomes of the second step, the individual chooses from the bundles of income and disposable income \((Y^i, I^i)\) offered by the government.\(^5\)

3.1.1 Step 2: Choice of commodity bundles

Given \(Y^i, I^i, q_c, q_d\), individuals of type \(i\) choose commodity bundles \((c^i_t, d^i_t)\) to maximize utility (1) subject to the annual budget constraint (4) and the credit constraint (2). The value function for this problem is:

\[
\psi^i(Y^i, I^i, q_c, q_d) = \max_{c^i_t, d^i_t} \sum_{t=1,2} u(c^i_t - \bar{c}, d^i_t) + \theta^i \left[ I^i - \sum_{t=1,2} (1 + r_i)^{t-1} (q_c c^i_t + q_d d^i_t) \right]
\]

\(^5\)For a similar approach, see Edwards et al. (1994)
$$- \mu^i \left[ q_c c^i_1 + q_d d^i_1 - \left( \frac{Y^i}{2} + \phi \right) \right],$$

where the credit constraint takes the values \( \phi \in \{0, \infty\} \), depending on the specific case under study. Applying the envelope theorem to the value function \( \psi^i(\cdot) \),

\[
\psi^i_I = \theta^i, \quad \psi^i_Y = \frac{\mu^i}{2}, \quad \psi^i_{q_c} = -\theta^i \sum_{t=1,2} (1 + r_i)^{t-1} c^i_t - \mu^i c^i_1, \quad \psi^i_{q_d} = -\theta^i \sum_{t=1,2} (1 + r_i)^{t-1} d^i_t - \mu^i d^i_1.
\]

Note that \( d\psi^i / d\phi = \mu^i \). Since consumer utility is non-decreasing in the size of the credit constraint \( \phi \), that implies \( \mu^i \geq 0 \) with the inequality applying when the constraint is binding. Note also that, by definition, \( \mu^i = 0 \ \forall i \) when \( \phi \to \infty \).

### 3.1.2 Step 1: Choice of income and net income bundles

Given commodity tax rates \((t_c, t_d)\) and anticipating step 2 above, the government on behalf of individuals of the two types offers income-consumption bundles \((Y^i, I^i)\). In an optimum, individuals choose the bundles intended for them. This yields total utility for a type-\( i \) person:

\[
V^i(Y^i, I^i, q_c, q_d) = \psi^i(Y^i, I^i, q_c, q_d) - h^i \left( \frac{Y^i}{w^i} \right).
\]

Using the envelope results (8) on \( \psi^i \), \( V^i(\cdot) \) satisfies the following properties:

\[
V^i_Y = \frac{\mu^i}{2} - \frac{1}{w^i} h^i \left( \frac{Y^i}{w^i} \right), \quad V^i_I = \psi^i_I, \quad V^i_{q_c} = \psi^i_{q_c}, \quad V^i_{q_d} = \psi^i_{q_d}.
\]

Preferences of an individual of type \( i \) in \((Y, I)\)-space have a slope:

\[
\frac{dI^i}{dY^i} = -\frac{V^i_I}{V^i_Y} = \frac{1}{\theta^i} \left[ \frac{1}{w^i} h^i \left( \frac{Y^i}{w^i} \right) - \frac{\mu^i}{2} \right]
\]

Finally, denote \( \hat{V}^i \) as the total indirect utility of a type \( i \) who mimics a type \(-i\). The mimicker will have the same income stream so will face the same credit constraint as the
individual being mimicked. Analogously to $V^i$ in (9), indirect utility is given by:

$$
\hat{V}^i(q_c, q_d, Y^{-i}, I^{-i}) = \psi^i(Y^{-i}, I^{-i}, q_c, q_d) - h\left(\frac{Y^{-i}}{w^i}\right).
$$  \hfill (11)

Similar envelope properties to (10) apply, and the slope of the mimicker’s indifference curves will be:

$$
d\hat{I}^i \frac{dY^i}{dI^i} = -\frac{\hat{V}^i}{\hat{V}^i_Y} - \frac{1}{\hat{V}^i_{Y^i}} \left[ \frac{1}{w^i} h'\left(\frac{\hat{Y}^i}{w^i}\right) - \frac{\mu^i}{2} \right].
$$

### 3.2 Tax implementation

Tax implementation involves finding marginal tax rates that implement the optimality conditions derived using mechanism design analysis. Doing so involves relating marginal tax rates to individual behavior as follows. The government implements a nonlinear tax function $T(Y^i)$. Using (3), we can rewrite the expression for indirect utility in (9) as

$$
V^i(\cdot) = \psi^i\left(Y^i, \left(\frac{2 + r_i}{2}\right) Y^i - T(Y^i), q_c, q_d\right) - h\left(\frac{Y^i}{w^i}\right).
$$

The individual chooses income $Y^i$ to maximize $V^i(\cdot)$. Using the envelope conditions (8), the first-order condition can be written

$$
\psi^i_Y + \psi^i_I \frac{\partial I^i}{\partial Y^i} - h_Y\left(\frac{Y^i}{w^i}\right) = \frac{\mu^i}{2} + \theta^i \cdot \left(\frac{2 + r_i}{2} - T'(Y^i)\right) - \frac{1}{w^i} h'\left(\frac{Y^i}{w^i}\right) = 0.
$$

Isolating the marginal tax rate gives

$$
T'(Y^i) = \frac{2 + r_i}{2} - \frac{1}{\theta^i w^i} h'\left(\frac{Y^i}{w^i}\right) + \frac{1}{2} \frac{\mu^i}{\theta^i}.
$$  \hfill (12)

This expression for $T'(Y^i)$ is the marginal tax wedge facing a type-$i$ individual. Below we use (12) to characterize the marginal tax rates that implement the solution to the government’s optimal tax problem.
3.3 Government’s problem

In our problem, the government redistributes from more productive to less productive individuals. We use the methodology developed by Hellwig (2007) — also applied by Bastani (2015) — to derive optimal tax schedules with a finitely large number of types. The government maximizes social welfare:

\[ W = \sum_i n_i \Phi(V_i) \]

subject to the budget constraint (6) and to \( N - 1 \) incentive compatibility (IC) constraints that take the form of downward adjacent constraints,

\[ V_i(Y_i, I_i, q_c, q_d; w_i) \geq \hat{V}_i(Y_{i-1}, I_{i-1}, q_c, q_d; w_i) \quad \forall i. \tag{\gamma_i} \]

where \( \Phi(V_i) \) is a concave social utility function, with \( \Phi'(V_i) > 0 \) and \( \Phi''(V_i) \leq 0 \). The function \( \hat{V}_i(Y_{i-1}, I_{i-1}, q_c, q_d; w_i) \) in the IC constraints is the indirect utility obtained by a type \( i \) who mimics the adjacent lower type \( i - 1 \), so is given by (11). The equation indicators \( \gamma_i \) represent the Lagrangian multipliers of the incentive constraints in the government’s problem, and \( \lambda \) is the Lagrangian multiplier for the budget constraint (6). Note that for \( R \) small enough, at least one type (the lowest) receives a transfer.

Given our assumption about preferences, all individuals would smooth their consumption across sub-periods 1 and 2 in the absence of credit constraints, albeit imperfectly. Credit constraints will be binding only for those expecting a transfer at the end of the period since then they will want to borrow in sub-period 1. Those who pay positive taxes will save at \( t = 1 \) to spread their tax liabilities across sub-periods.

We consider the government problem in three successive settings of increasing complexity. We begin with the benchmark case in which no one is credit-constrained and there is no credit spread. We then assume a credit constraint with \( \phi = 0 \) that is binding on at least one type (the lowest), but restrict the government to using a nonlinear income tax. The
credit spread is irrelevant in this case since no individuals will borrow, and the government gets all its revenues at the end of the period. In the final case, both credit constraints and a credit spread apply, and we let the government choose differentiated commodity taxes or subsidies alongside the nonlinear income tax. We denote by $\mathcal{L}(\cdot)$ the Lagrangian function of the government. The first-order conditions for the government’s problem in the third, most general, setting where the credit constraint is binding for at least one type and the government chooses commodity tax rates are listed in Appendix A.

3.3.1 Benchmark case: unconstrained individuals and no credit spread

This case corresponds to the standard optimal nonlinear income tax problem with linear commodity taxes. All individuals and the government can borrow and lend at the common interest rate $r$. First, we establish that the government need not use commodity taxation at all, and then we characterize the optimal income tax system.

The government can normalize commodity taxes by setting $q_c = 1$, and then optimize on relative commodity prices which we denote by $\alpha = q_d/q_c$. Choosing $\alpha$ is equivalent to choosing $q_d = 1 + t_d$. Since individuals are not credit-constrained, $\mu^i = 0$, in the individual’s value function (7). Using the envelope properties for the individuals in (8) and (10), the government’s first-order conditions shown in Appendix A lead to the standard Atkinson-Stiglitz theorem:

Proposition 1. When $i = 1, ..., N$ are unconstrained and there is no credit spread, then the Atkinson-Stiglitz Theorem holds and commodity taxes are undifferentiated.

Proof: See Appendix B.

Thus, the Atkinson-Stiglitz theorem continues to apply even though consumption and labor supply occur sequentially over the tax year. The theorem stipulates only that commodity taxes should be uniform if used, but since uniform commodity taxes are equivalent to
a proportional income tax in this benchmark case, they are redundant and thus unnecessary.

Since credit constraints are not binding in this case, \( \mu^i = 0 \), so the marginal tax wedge in (12) simplifies to

\[
T'(Y^i) = \left( \frac{2 + r}{2} - \frac{1}{\theta^i w^i} h'\left( \frac{Y^i}{w^i} \right) \right).
\]

(13)

Using the first-order conditions in Appendix A to rewrite the right-hand side of (13), we obtain the following marginal tax formulas in an optimum:

\[
T'(Y^i) = \frac{\theta^i \gamma^{i+1}}{\lambda n_i} \left( \frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right).
\]

(14)

As shown by Hellwig (2007), the term in parentheses is always positive when the single-crossing condition is satisfied and when leisure is an normal good. Moreover, \( \gamma^{N+1} = 0 \) since there is no downward incentive constraint at the top. Therefore, marginal tax rates are everywhere positive except at \( Y^N \), for which \( T'(w^N) = 0 \) so there is no distortion. These are the standard optimal income tax results.

### 3.3.2 Case with binding credit constraints and no commodity taxes

Suppose now that transfers to the lowest types are sufficiently large that the credit constraint on at least one type is binding, so \( \mu^i > 0 \) for at least some \( i \). Those whose credit constraint does not bind pay taxes at the end of the period and save for it, whereas the poorest ones would have liked to borrow using future transfers as collateral but they cannot. There are no commodity taxes in this specific case, so \( q_c = q_d = 1 \). As a consequence, the government has no revenues and no expenses in the first sub-period, and \( r_g \) does not need to be specified. Those who save do so at rate \( r_i = r \).

To characterize the optimal income tax system, note first that the tax wedge (12) can
now be written
\[ T'(Y^i) = \left( \frac{2 + r}{2} \right) - \frac{1}{\theta^i w^i} h^i \left( \frac{Y^i}{w^i} \right) + \frac{1}{2} \frac{\mu^i}{\theta^i}. \]

For those who are not credit-constrained, \( \mu^i = 0 \) as above. For credit-constrained lower-income individuals, \( \mu^i > 0 \). They will be more inclined to work more to generate income in the first-period.

As above, we can use the government’s first-order conditions from Appendix A to obtain optimal marginal income tax rates. They take exactly the same form as the standard case above given by (14). However, compared with the benchmark case, this will result in lower marginal tax rates for the credit-constrained individuals for two main reasons. First, binding credit constraints lower utility, and this reduces the incentive of higher types to mimic, which has an effect on \( \gamma^{i+1} \). consumption in the first period.

3.3.3 Case with binding credit constraints and income and commodity taxes

When the government has access to commodity taxes or subsidies, it can use them to relax the binding incentive constraints by subsidizing consumption in sub-period 1. But, this comes at a cost since it must borrow at the rate \( r_g \) to finance the subsidies. Given that commodity subsidization also benefits the unconstrained individuals, saving of the latter is increased. Since \( r_g > \bar{r} \), government saving accompanied by private dissaving results in a resource cost.

In this case, two outcomes are possible. First, subsidization of commodities may be sufficient to eliminate the credit constraint of all low-income individuals. Alternatively, the cost of commodity subsidization may be sufficiently large that in an optimum, some low-income individuals remain credit-constrained. Since the qualitative results differ in the two cases, we consider them separately.

Case A. Credit constraints eliminated in the optimum

In this case, which happens when \( r_g - \bar{r} \) is small enough, the government can use commodity
taxes to undo the credit constraint of all individuals. The following proposition is proved in Appendix B.

**Proposition 2:** If in the optimum $\mu^i = 0, \forall i$, so policy relaxes all credit constraints in the economy, then $t_{c}^{*} = t_{d}^{*} < 0$.

Thus, both goods are subsidized at the same rate. The commodity subsidy system acts as a proportional subsidy on income. Since credit constraints are not binding, the equivalent of a second-best is recovered, although with interest rates higher for borrowers (the government) than for savers.

To ensure that the entire tax system maximizes social welfare in an incentive-compatible way, income tax rates are adjusted to reflect the fact that uniform commodity subsidies are equivalent to a subsidy on income. In the optimum, the effective marginal income tax rate of individual $i$ taking account of both income tax and commodity subsidy distortions is identical to the one obtained in the benchmark case without credit constraints and commodity subsidies.

In particular, the marginal tax rate faced by individual $i$ is now

$$T'(Y^i) = \frac{\theta^i \gamma^i + 1}{\lambda n_i} \left( \frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right) - t^* \sum_t (1 + r_g)^{2-t} \left( z_i - T'(Y^i) \right) \left( \frac{\partial c^i_t}{\partial I^i} + \frac{\partial d^i_t}{\partial I^i} \right),$$

where $t^* \equiv t_{c}^{*} = t_{d}^{*}$ and $z_i = (2 + r_i)/2$. This tax formula, analogous to that derived by Edwards et al. (1994), shows that when the government subsidizes consumption proportionally, this creates purchasing power that is identical to an increase in net income. Therefore, a share $(z - T'(Y^i))$ of the “income value” of the subsidy has to be left in the individuals’ pockets, adjusted for the funding cost of the subsidies $r_g$.

The tax formula in (15) reflects the fact that the wedge between labor and consumption must encompass the marginal incentives and disincentives generated by all tax instruments.
This can be seen by rewriting (15) as a marginal effective tax rate:

\[
T'(Y^i) + t^* \sum_t (1 + r_g)^{2-t}(z_i - T'(Y^i)) \left( \frac{\partial c^i_t}{\partial I^i} + \frac{\partial d^i_t}{\partial I^i} \right) = \frac{\theta^i \gamma^{i+1}}{\lambda n_i} \left( \frac{h'(Y^i/w^i)}{\theta^i w^i} - \frac{h'(Y^i/w^{i+1})}{\theta^i w^{i+1}} \right),
\]

(16)

where the righthand side is the optimal labor wedge in (14). This shows that subsidies on consumption goods must be clawed back by increases in marginal income tax rates. It also shows that the standard properties of the optimal tax systems are not violated, including positive marginal tax rates at all income levels and a zero effective marginal tax rate at the top.

Finally, note that the most extreme case of such an optimum would be when subsidies can be funded at no opportunity cost for the government, or when \( r_g = r \). Then, transferring purchasing power from the second to the first period is done at no cost for the government, and it can always lower the prices of commodities so as to make all credit constraints in the economy slack. In this unrealistic example, the timing of income-tested payments has no effective consequence on the overall tax policy and social optimum. As soon as subsidies become costly, there is a threshold level of the cost beyond which the government will leave some individuals credit constrained.

**Case B. Some credit constraints binding in the optimum**

When the cost of funding commodity subsidies in the first period are high enough, it may be optimal for the government to leave some individuals’ credit constraints binding. When this happens, differential subsidy or tax rates should apply, but it may be optimal either to subsidize both goods, or to subsidize one and tax the other.

The intuition is as follows. Commodity taxation generates both income and substitution effects. With no binding credit constraints, the nonlinear tax system can adjust to offset income effects, and this ensures that it is optimal not to create substitution effects. With
binding credit constraints, for those who are constrained in the optimum some income effects cannot be clawed back by proportional adjustments in the income tax schedule. These income effects are costly for the government because they induce an increase in consumption in the first period which is subsidized and must be financed through short-term debt. On the other hand, subsidizing at least one commodity is beneficial because it helps reducing the pressure of credit constraints. Thus, the government needs to compromise and “spend” its resources on the good that is proportionately more consumed by the constrained.

As it turns out, this may happen even when utility-of-consumption functions \( u(\cdot) \) feature linear Engel curves. Our simple case in which there is a basic need for good \( c, \bar{c} \), justifies either subsidizing good \( c \) at a higher rate, or simply subsidizing \( c \) and taxing \( d \) if \( r_g - r \) is very high. These results are illustrated in the numerical section below. Proposition 3, whose proof is given in Appendix B, gives the general condition under which differentiation happens under linear Engel curves.

**Proposition 3:** Let us denote \( B_i \equiv (\mu_i / \lambda)(\Phi'(V^i) + \gamma i / n_i - \gamma i+1 / n_i) - (r_g - r_i) \). Also denote by \( \mathcal{C} \neq \emptyset \) the subset of types whose credit constraint bind in the optimum: \( i \in \mathcal{C} \Leftrightarrow \mu_i > 0 \). Then, the optimal policy has \( t_d > t_c \) if and only if

\[
\left( \frac{\sum_i n_i B_i c^i_i}{\sum_{i \in \mathcal{C}} n_i c^i_1} - \frac{\sum_i n_i B_i d^i_i}{\sum_{i \in \mathcal{C}} n_id^i_1} \right) > 0.
\]

To understand the intuition contained in the proposition, let us consider the following expression:

\[
n_i B_i \equiv (\mu_i / \lambda)(n_i \Phi'(V^i) + \gamma i - \gamma i+1) - n_i(r_g - r_i).
\]

This is the marginal social valuation, accounted for in dollars, associated with giving one extra lump-sum dollar to all type-\( i \) individuals in the first period, while the second-period income tax schedule adjusts to remain optimal. It includes the marginal social welfare weight of individual \( i \) and the marginal effects on mimicking (adjacent, below and above) if he is
constrained, and the mechanical cost on the budget constrained associated with funding the dollar through debt. The term $B_i$ is necessarily negative for $i \notin C$ since the income effects generated by commodity subsidization are repaid for by an adjustment in $T(Y^i)$, except for the net cost $-n_i(r_g - r)$ related with funding $n_i$ dollars of subsidies. However, it can take either sign if $i \in C$, and it is presumably declining when $w^i$ increases, since the multiplier on the credit constraint, $\mu^i$, declines with it and because the social welfare function is concave.\(^6\)

The terms $\sum_{i \in C} n_i c_i$ and $\sum_{i \in C} n_i d_i$ are the costs for the government that are associated with income effects that remain for constrained individuals. When a dollar of purchasing power is given in the first period to constrained individuals through indirect taxation, these individuals increase their consumption of both goods in the first period due to the income effect. The more one good is consumed, the costlier this income effect is for the government. This income effect does not show up for unconstrained individuals, since following an increase in a subsidy on a good, the government can adjust their second-period income tax schedule to ‘repay the subsidy’ and no income effective income effect is then experiences.

Overall, Proposition 3 states that the optimal policy involves differentiating commodity taxes, with $t_c < t_d$, if the optimal policy involves a higher benefit/cost ratio of subsidizing good $c$ than good $d$. Corrolary 1, which is proven below, states that if Engel’s curves are linear and no basic need, then it is optimal to not differentiate commodity tax rates. This result is independent of the shape of the social welfare function $\Phi(\cdot)$. The intuition is that the government wants to subsidize more a good that is proportionately consumed more by constrained individuals. When $\bar{c} = 0$, all individuals consume goods $c$ and $d$ in same proportions, and the government decides not to generate substitution effects. Interestingly, the Corrolary implies that credit constraints alone, or the presence of a basic need alone, are not sufficient to justify differentiation.

\(^6\)A sufficient condition for $n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1} > 0$ is that $\max\{t_c, t_d\} < (1 + r_c)/(1 + r_g)$ when both goods are normal. This condition, which right-hand side is larger than 1, is obtained by decomposing equation (22) and applying Walras’s Law.
Corollary 1: If $\bar{c} = 0$, then there is no differentiation and $t^*_c = t^*_d < 0$. This result is independent of the specific social welfare function $\Phi(\cdot)$ used by the social planner.

Proof: Denote by $\alpha_i \equiv c^i/d^i$ in the optimum. Since Engel curves are linear and affine through the origin, $\alpha_i = \alpha, \forall i$. Thus,

$$ \left( \frac{\sum_i n_i B_i c^i}{\sum_i n_i B_i d^i} - \frac{\sum_{i \in C} n_i c^i}{\sum_{i \in C} n_i d^i} \right) = \left( \alpha \frac{\sum_i n_i B_i d^i}{\sum_i n_i B_i d^i} - \alpha \frac{\sum_{i \in C} n_i d^i}{\sum_{i \in C} n_i d^i} \right) = 0. $$

This result is independent of the specific form of $\Phi(\cdot)$, which only appears in $B_i$, $\forall i$. ■

4 Numerical examples

We illustrate our results with some numerical simulations. The quasi-homothetic utility function and disutility of labor are respectively

$$ u(c^i, d^i) = \kappa \frac{(c^i - \bar{c})^{1-\rho}}{1-\rho} + \kappa \frac{d_i^{1-\rho}}{1-\rho}; \quad (18) $$

$$ h(l) = \frac{l^{1+\sigma}}{1+\sigma}, \quad l \equiv Y/w. \quad (19) $$

We set the values of the parameters to $\rho = 0.9$ and $\sigma = 2$. The constant $\kappa$ is chosen so the solution of the optimal tax problem gives the same utility level and optimal tax function as in the fictive case where $\bar{c} = 0$ with $r = 0$, and where the yearly utility of an individual would be $u(c, l) = c^{1-\rho}/1-\rho - l^{1+\sigma}/1+\sigma$. Its value is therefore equal to $(1/4)^\rho$.

The number of workers at each wage level, $n_i$, is obtained using a lognormal distribution with parameters $(\mu, \sigma) = (2.757, 0.5611)$. It is explicitly taken from Mankiw et al. (2009), who estimate these parameters from the 2007 March wave of the Current Population Survey (CPS). We then discretize the distribution to obtain 100 wage levels with a fixed distance.
between any two wage levels. The probability mass function is rescaled so that \( \sum_i n_i = 1 \). We initially set the interest rate spread to 15 percentage points (which is a bit lower than borrowing rates on credit cards), with \( r = 0 \) and \( r_g = 0.15 \), although sensitivity analyses are also computed.

We use the basic need \( \bar{c} = 5 \), which approximately represents a $5.75 daily, or $2,100 annually in our model. This amount appears to us as being fairly conservative. For instance, the average Supplemental Nutrition Assistance Program (SNAP), commonly referred to as the “food stamp” program, is about $4 a day. Moreover, the 2007 Federal poverty line is set at $10,210 ($ 28 per day) for a single-person household. The year 2007 is used to calibrate our simulations, in order to make them comparable with Mankiw et al. (2009).

Due to the introduction of commodity taxes and subsidies, we report the effective tax burden paid at a given level of income, and the effective marginal tax rates as in Edwards et al. (1994). The total tax burden of a worker at labor income \( Y^i \), in the eyes of the government, is

\[
T^E(Y^i) = T(Y^i) + t_c \left[ \sum_t (1 + r_g)^{2-t} c^i_t \right] + t_d \left[ \sum_t (1 + r_g)^{2-t} d^i_t \right].
\] (20)

We also report marginal effective tax rates as derived in equation (16).

Baseline simulations

We start considering three scenarios. The first one is the \textit{No credit constraint} scenario. Individuals can save and borrow at the same rate. This first scenario is a useful benchmark as it gives the same optimal allocation and effective marginal tax rates as when the planner can subsidize first-period consumption at no cost. In this sense, this is analogous to a standard second-best tax regime. In the second scenario (\textit{Credit constraint}) individual’s borrowing limit is \( \phi = 0 \). The planner resorts to nonlinear income taxation to achieve his redistribu-
tive objective. With $r = 0$, individuals are constrained whenever they expect receiving a
transfer at the end of the fiscal year ($T(Y^i) < 0$). Commodity taxes and subsidies are as-
sumed away. The third scenario (Credit constraint with subsidies) introduces commodity
taxation/subsidization. As a benchmark, we start using an utilitarian social welfare function
($W = \sum_i n^i V^i$), followed by other examples featuring Pareto weights.

[Figure 1 about here]

The characteristics of the optimal tax systems for all three scenarios are shown in Figure 1 and in Table 1. From Figure 1, it is possible to see that the Credit constraint and subsidies scenario has the highest marginal tax rates, except at the very bottom of the income
distribution, and also the highest average tax rates. The optimal solution to this scenario
where the government has access to commodity taxation will feature subsidies to consump-
tion goods. To finance these subsidies the planner must raise revenues using of labor income
taxes.

Looking at the optimal effective marginal tax rates, one sees that the Credit constraint
and subsidies scenario is an intermediate case, where only the nonlinear labor income tax
available as a policy instrument. The No credit constraint scenario features the highest
effective marginal tax rates. As is typical with second-best optimal income tax models, in
the absence of credit constraint the government heavily distorts labor supply at the bottom
of the distribution, and the marginal effective (income) tax rates decline when we move
towards the top of the skills distribution. Contrastingly, the Credit constraint case, which
precludes the use of subsidies, has the lowest marginal effective tax rates for all the skills
distribution. Since there is a basic need $\bar{c} > 0$, low-skill workers must supply more labor in
order to meet their basic need, the planner being unable to transfer resources to them in the
first period. Moreover, the inability of low skilled workers to smooth consumption reduces
the redistributive potential of the income tax schedule, thereby reducing the need to collect
taxes for redistributive purposes.

25
The Credit constraint with subsidies scenario allows the planner to transfer resources in the first period using indirect taxation, albeit at a cost, to the first period through subsidies on goods. The same qualitative pattern as with the scenario precluding indirect taxation emerges at the bottom of the income distribution because the planner does not want to discourage work. When goods are subsidized, labor income gives a higher marginal utility since subsidies increase the purchasing power of each additional dollar earned early in the fiscal year.

Table 1 shows that, through subsidization, fewer workers are constrained if commodity taxation is a policy tool. The utilitarian social welfare levels are reported for all three scenarios. That with credit constraints and commodity taxes/subsidies dominates the scenario with credit constraints but no commodity taxation. The table also presents an alternate measure of social welfare, which is, a calculation of the welfare gains starting from the laisser-faire allocation. This gain interpreted as the minimal percentage increase in consumption from the laisser-faire, required to attain the same welfare levels as in the relevant scenarios (\(\%\Delta^{LF}\)). This percentage is fixed across individuals and time-periods.\(^7\). Once more, we observe that the case with commodity taxes is preferred to that without. The small differences are due to the utilitarian social welfare preference and the limited curvature of the consumption utility function, which puts less weight on the lower skilled workers.

**Putting more weight on the poor**

We now use a weighted utilitarian social welfare function with weights

\[
\omega_i \equiv \frac{w_i^{-\eta}}{\sum_i n_i w_i^{-\eta}}, \quad \eta \geq 0; \quad \sum_i n_i \omega_i = 1
\]

\(^7\)This way to account for welfare variations is documented in Farhi & Werning (2013) and ?
where the social welfare is given by \( W \equiv \sum_i n^i \omega_i V^i \). This function puts more weight on workers with low wages as \( \eta \) increases. Then, allowing the government to subsidize goods will capture most of the gains from the No credit constraint scenario, measured again by the percentage increase in consumption from the laissez-faire allocation. Table 2 reports the welfare gains of the Credit constraint and the Credit constraint with subsidies scenarios. The more the government cares about the low-wage individuals, the less palatable it becomes to leave them credit constrained. This means that relaxing their credit constraint using indirect taxation becomes a priority for the government.

[Table 2 about here]

Changes in basic need

We consider changes in the level of basic need \( \bar{c} \). Figure 4 shows that an increase in the basic need induces the social planner to subsidize consumption more. It also induces him to differentiate more the tax rates. We can see that with \( \bar{c} = 0 \) the planner will still wish to subsidize consumption, but that relative prices remain undistorted. The greater the basic need the harder and costlier it is in to achieve that basic level of consumption through work effort. Hence, the government wants to use increasingly more resources at helping low wage workers by increasing the value of their labor earnings. This involves subsidizing more heavily the goods that are proportionately consumed more by the credit constrained individuals.

[Figure 4 about here]

Figure 5 illustrates the effect of an increase in the basic need on labor income taxes. The planner requires higher income tax levels to finance the subsidies. This is visible from the

\footnote{For every different levels of \( \eta \), new simulations are computed where the optimal policies were chosen with the new social welfare as the government’s objective.}
plots depicting marginal and average tax rates. The higher basic need also increases the level of redistribution, which results in higher effective marginal tax rates for all workers, but especially at the bottom of the income distribution. Average effective tax rates also increase with \( \bar{c} \), since a higher basic need increases marginal utility of consumption of the necessity good at a given level of consumption. This reinforces the redistributive motive of the government.

\[ \text{Figure 5 about here} \]

**Changes in interest rates**

We finish this section by exploring changes in the credit spread, \( r_g - r \). To focus on the changes in the costs and not on increases in the amount of resources available to the economy due to higher interest rates on savings, we only change the level of \( r_g \), keeping \( r \) = 0. Figure 2 presents the same information in three different manners. The main point is that reducing the government’s cost of borrowing bolsters its incentive to subsidize commodities, and reduce its willingness to differentiate the rates of these subsidies. If the cost is high enough, we eventually obtain a positive tax rate the non-necessity good \( d \) and a tax on \( c \). The planner wants to use every costly resource available to ensure that all individuals consume at least \( \bar{c} \).

Figure 3 shows the effect of different costs on the optimal labor income tax schedule. Whenever costs to subsidize are low, the optimal tax systems features large subsidies and requires higher labor income taxation as can been seen by both the marginal tax rates and average tax rates. As above, when the planner is able to transfer more money in the first period, he is also able to distort the labor decision of lower skilled workers a bit more and attempt more redistribution. This can be seen by looking at the effective marginal tax rates. The case with the highest spread, and thus the highest costs, also features the lowest effective
marginal income tax rates since the planner must encourage work especially at the bottom of the distribution.

\[\text{[Figure 3 about here]}\]

\[\text{[Figure 2 about here]}\]

5 Conclusions and an extension to lump-sum transfers

In this paper, we studied an optimal tax system when transactions on goods happen more frequently than the payment of income-tested transfers. Credit constraints arise because individuals cannot fully use future transfers as collateral. Our results show that when the optimal policy is able to relax the constraint on all individuals, it involves proportional subsidies on all goods. When the cost of providing the subsidies is too high, differentiation may happen when constrained individuals spend a higher proportion of their disposable income on a good (for instance a necessity) than the general population. Then, the government can either subsidize all goods at differential rates, or subsidize some commodities while taxing others.

As mentioned in the Introduction, if the government could make lump-sum transfers in sub-period 1 separately from its income tax-transfer system at the end of the period, it could use them to ameliorate binding credit constraints. This opens the door to interesting interactions between commodity taxation and these payments.

Consider first the extreme case where \( r_g = r = \bar{r} \), so the lump-sum transfers could be made costlessly by the government borrowing against future tax revenues. The government will offer a universal lump-sum transfer at \( t = 1 \) sufficiently large to relax all credit constraints and reduce all tax liabilities at the end of the period by an equivalent amount. If this scheme
is available to the planner, then the allocation would be identical to our benchmark case without credit constraints. There would be no need to use commodity subsidies and the Atkinson-Stiglitz Theorem would hold.

Suppose instead that $r_g > r$ so the government faces a cost of transferring money from the second to the first sub-period. In this scenario, the government may no longer be able to front-load completely redistribution in the first sub-period and adjust the income tax schedule to leave all workers unconstrained. If the government uses only nonlinear income taxation along with the universal lump-sum transfer, the situation is similar to our second case above. Some individuals remain credit-constrained, and the standard form of the optimal marginal income tax rates but since some individuals remain credit-constrained social welfare is less than in the benchmark case.

When the government can use commodity taxes along with period-1 lump-sum transfers, things change potentially dramatically. Uniform commodity taxes combined with a lump-sum transfer is equivalent to a linear progressive tax system which transfers income from high- to low-income individuals. Moreover, the commodity taxes generate tax revenue for the government to finance the lump-sum transfers thereby offsetting the need to borrow. If the government sets the commodity tax rate high enough, it would seem it could finance an amount of lump-sum transfers sufficient to relax all credit constraints of low-income individuals while at the same time avoiding the need to borrow. And, if the utility-of-consumption function $u(\cdot)$ satisfies the Deaton conditions, it would seem to be optimal to use undifferentiated commodity taxes.

In fact, the optimal allocation achieved in this manner appears likely to lead to more credit-constrained workers. This is because the high level of commodity taxes must apply in both periods. This induces the government to adjust the labor income tax schedule to make the allocation incentive compatible. To do this, the tax burden of workers is drastically reduced to the point where a majority of workers face net transfers from the labor income tax. This leads many workers to want to borrow but are unable to do so due to the credit
constraint.

We conjecture that the uniform commodity tax result obtained will break down if Engel curves are nonlinear. Furthermore, the ability of the government to use commodity tax revenues obtained from period-1 commodity purchases to finance the lump-sum transfer requires that the government actually receive those revenues in period 1. In practice, the firms collecting commodity taxes will not remit them to the government until the end of the tax year and this will cause the above mechanism to break down. Even in the case if linear Engel curves, we conjecture that divorcing the timing of the collection of commodity tax revenues from the payment of the lump-sum transfer will also lead to differentiated commodity taxes. In fact, we should be able to recover many of the results found earlier in this paper. Proving these conjectures is left to further research.
References


A First-order conditions of the general problem

The Lagrangian of the government is

\[ \mathcal{L} = \sum_{i=1}^{N} n_i \Phi(V^i) + \sum_{i=2}^{N} \gamma^i [V^i(Y^i, I^i, q_c, q_d; w^i) - \hat{V}^i(Y^{i-1}, I^{i-1}, q_c, q_d; w^i)] \]

\[ + \lambda \left[ \sum_{i=1}^{N} n_i \left( \left( \frac{2 + r_i}{2} \right) Y^i - I^i \right) + (q_c - 1) \sum_{i=1}^{N} n_i \sum_t (1 + r_g) 2^{-t} c^i_t \right. \]

\[ \left. + (q_d - 1) \sum_{i=1}^{N} n_i \sum_t (1 + r_g) 2^{-t} d^i_t \right] . \]

We present the first-order conditions in their most general form to keep the notation as compact as possible. Note that, by the definition of the problem, \( \gamma^1 \equiv 0 \) since the lowest type cannot mimic any lower adjacent type. Note, also, that all types that are not constrained in the optimum have \( \mu^i = 0 \). Those who are constrained have \( \partial c^i_1 / \partial I^i = \partial d^i_1 / \partial I^i = 0 \) and those who are not constrained have \( \partial c^i_1 / \partial Y^i = \partial d^i_1 / \partial Y^i = 0 \). The first-order conditions are:

\[ \frac{\partial \mathcal{L}}{\partial I^i} = n_i \Phi'(V^i) \theta^i + \gamma^i \theta^i - \gamma^{i+1} \theta^i - \lambda n_i + \lambda n_i (q_c - 1) \sum_t \frac{\partial c^i_t}{\partial I^i} (1 + r_g)^{2-t} \]

\[ + \lambda n_i (q_d - 1) \sum_t \frac{\partial d^i_t}{\partial I^i} (1 + r_g)^{2-t} = 0 \quad \forall i, \quad (22) \]

\[ \frac{\partial \mathcal{L}}{\partial Y^i} = n_i \Phi'(V^i) \left( \frac{\mu^i}{2} - \frac{1}{w^i} h' \left( \frac{Y^i}{w^i} \right) \right) + \gamma^i \left( \frac{\mu^i}{2} - \frac{1}{w^i} h' \left( \frac{Y^i}{w^i} \right) \right) \]

\[ - \gamma^{i+1} \left( \frac{\mu^i}{2} - \frac{1}{w^{i+1}} h' \left( \frac{Y^i}{w^{i+1}} \right) \right) + \lambda \left( \frac{2 + r}{2} \right) n_i \]

\[ + \lambda n_i (q_c - 1) \sum_t \frac{\partial c^i_t}{\partial Y^i} (1 + r_g)^{2-t} + \lambda n_i (q_d - 1) \sum_t \frac{\partial d^i_t}{\partial Y^i} (1 + r_g)^{2-t} = 0 \quad \forall i, \quad (23) \]
\[
\frac{\partial L}{\partial q_c} = \sum_i n_i \Phi'(v^i) \left( -\theta_i \sum_t (1 + r_i)^{2-t}c^i_t - \mu^i c^i_1 \right)
+ \sum_{i=1}^{N-1} (\gamma^i - \gamma^{i+1}) \left( -\theta^i \sum_t (1 + r_i)^{2-t}c^i_t - \mu^i c^i_1 \right)
+ \lambda (q_c - 1) \sum_i n_i \sum_t \frac{\partial c^i_t}{\partial q_c} (1 + r_g)^{2-t} + \lambda (q_d - 1) \sum_i n_i \sum_t \frac{\partial d^i_t}{\partial q_c} (1 + r_g)^{2-t}
+ \lambda \sum_i n_i \sum_t c^i_t (1 + r_g)^{2-t} = 0. \tag{24}
\]

\[
\frac{\partial L}{\partial q_d} = \sum_i n_i \Phi'(v^i) \left( -\theta_i \sum_t (1 + r_i)^{2-t}d^i_t - \mu^i d^i_1 \right)
+ \sum_{i=1}^{N-1} (\gamma^i - \gamma^{i+1}) \left( -\theta^i \sum_t (1 + r_i)^{2-t}d^i_t - \mu^i d^i_1 \right)
+ \lambda (q_c - 1) \sum_i n_i \sum_t \frac{\partial c^i_t}{\partial q_d} (1 + r_g)^{2-t} + \lambda (q_d - 1) \sum_i n_i \sum_t \frac{\partial d^i_t}{\partial q_d} (1 + r_g)^{2-t}
+ \lambda \sum_i n_i \sum_t d^i_t (1 + r_g)^{2-t} = 0. \tag{25}
\]

**Effective marginal tax rates**

By (22), obtain \forall i

\[
(n_i \Phi'(v^i) + \gamma^i - \gamma^{i+1}) \theta^i = \lambda n_i \left( 1 - t_c \sum_t \frac{\partial c^i_t}{\partial I^i} (1 + r_g)^{2-t} - t_d \sum_t \frac{\partial d^i_t}{\partial I^i} (1 + r_g)^{2-t} \right). \tag{26}
\]

Using \(z_i \equiv (2 + r_i)/2\), \(z_i - T' = \frac{h'(Y^i/w^i)}{w_i \theta^i} - \frac{\mu^i 1}{\theta^i 2} \), and adding and subtracting

\[
\gamma^{i+1} \theta^i \left( \frac{\mu^i 1}{\theta^i 2} - \frac{h'(Y^i/w^i)}{w^i \theta^i} \right)
\]

35
in (23), obtain
\[ (n_i \Psi'(V^i) + \gamma^i - \gamma^{i+1}) \theta^i (T'(Y^i) - z_i) - \gamma^{i+1} \theta^i \left( \frac{h'(Y^i/w^i)}{w^i \theta^i} - \frac{h'(Y^i/w^{i+i})}{w^{i+i} \theta^i} \right) \]
\[ + \lambda n_i \left( z_i + t_c \sum_t \frac{\partial c^i_t}{\partial Y^i} (1 + r_g)^{2-t} + t_d \sum_t \frac{\partial d^i_t}{\partial Y^i} (1 + r_g)^{2-t} \right) = 0. \] (27)

Substituting (26) into (27) and reorganizing, one obtains
\[ T'(Y^i) + t_c \sum_t (1 + r_g)^{2-t} \left( \frac{\partial c^i_t}{\partial Y^i} (z_i - T') + \frac{\partial c^i_t}{\partial I^i} \right) \]
\[ + t_d \sum_t (1 + r_g)^{2-t} \left( \frac{\partial d^i_t}{\partial Y^i} (z_i - T') + \frac{\partial d^i_t}{\partial I^i} \right) \]
\[ = \frac{\gamma^{i+1} \theta^i}{\lambda n_i} \left( \frac{h'(Y^i/w^i)q_c}{w^i \theta^i} - \frac{h'(Y^i/w^{i+i})}{w^{i+i} \theta^i} \right). \] (28)

### B Proofs

For further use, we present the expressions used for compensated demands. We compensate demands by varying disposable income \( I^i \) but taking annual earnings \( Y^i \) as given. For an unconstrained individual and using a tilde to denote compensated demands, the Slutsky equations can be written

\[ \frac{\partial c^i_t}{\partial q_c} = \frac{\partial \tilde{c}^i_t}{\partial q_c} - \frac{\partial c^i_t}{\partial I^i} \sum_{t=1,2} (1 + r_i)^{2-t} c^i_t, \quad \frac{\partial d^i_t}{\partial q_c} = \frac{\partial \tilde{d}^i_t}{\partial q_c} - \frac{\partial d^i_t}{\partial I^i} \sum_{t=1,2} (1 + r_i)^{2-t} c^i_t, \] (29)

\[ \frac{\partial d^i_t}{\partial q_d} = \frac{\partial \tilde{d}^i_t}{\partial q_d} - \frac{\partial d^i_t}{\partial I^i} \sum_{t=1,2} (1 + r_i)^{2-t} d^i_t, \quad \frac{\partial c^i_t}{\partial q_d} = \frac{\partial \tilde{c}^i_t}{\partial q_d} - \frac{\partial c^i_t}{\partial I^i} \sum_{t=1,2} (1 + r_i)^{2-t} d^i_t. \] (30)

If an individual is constrained, then in the first period \( \partial c^i_1/I^i = \partial d^i_1/I^i = 0 \). Given time-separability we can make use of the fact that first-period expenditures satisfies \( q_c c^i_1 + q_d d^i_1 = \)
\[ Y^i/2 + \phi, \text{ evaluated locally at } \phi = 0. \] Then, compensated demands at \( t = 1 \) satisfy

\[ \frac{\partial c^i_1}{\partial q_c} = \frac{\partial c^i_1}{\partial q_c} - c^i_1 \frac{\partial c^i_1}{\partial \phi}, \quad \frac{\partial d^i_1}{\partial q_c} = \frac{\partial d^i_1}{\partial q_c} - c^i_1 \frac{\partial d^i_1}{\partial \phi}, \] (31)

\[ \frac{\partial d^i_1}{\partial q_d} = \frac{\partial d^i_1}{\partial q_d} - d^i_1 \frac{\partial d^i_1}{\partial \phi}, \quad \frac{\partial c^i_1}{\partial q_d} = \frac{\partial c^i_1}{\partial q_d} - d^i_1 \frac{\partial c^i_1}{\partial \phi}. \] (32)

Therefore, following a marginal change in one price, keeping labor effort constant, compensation can be achieved by allowing the constrained individual to borrow marginally more.

In the second period,

\[ \frac{\partial c^i_2}{\partial q_c} = \frac{\partial c^i_2}{\partial q_c} - \frac{\partial c^i_2}{\partial I^i c^i_2}, \quad \frac{\partial d^i_2}{\partial q_c} = \frac{\partial d^i_2}{\partial q_c} - \frac{\partial d^i_2}{\partial I^i c^i_2}, \] (33)

\[ \frac{\partial d^i_2}{\partial q_d} = \frac{\partial d^i_2}{\partial q_d} - \frac{\partial d^i_2}{\partial I^i d^i_2}, \quad \frac{\partial c^i_2}{\partial q_d} = \frac{\partial c^i_2}{\partial q_d} - \frac{\partial c^i_2}{\partial I^i d^i_2}. \] (34)

**Proposition 1:** When \( i = 1, \ldots, N \) are unconstrained and there is no credit spread, then the Atkinson-Stiglitz Theorem holds and commodity taxes are undifferentiated.

**Proof:** First, define the price ratio \( \alpha \equiv q_d / q_c \). Set \( q_c = 1 \), so the first-order condition (25) chooses \( \alpha \) and (24) can be ignored. Take the first-order conditions with respect to \( I^i \) in (22), multiply them by \( \sum_t (1 + r)^{2-t} d^i_t \), to obtain

\[ n_i \Phi'(V^i) \theta^i \sum_t (1 + r)^{2-t} d^i_t + (\gamma^i - \gamma^{i+1}) \theta^i \sum_t (1 + r)^{2-t} d^i_t - \lambda n_i \sum_t (1 + r)^{2-t} d^i_t \]

\[ + \lambda n_i (\alpha - 1) q_c \sum_t \frac{\partial d^i_t}{\partial I^i} (1 + r)^{2-t} \sum_t (1 + r)^{2-t} d^i_t = 0 \quad \forall i, \]

37
Rearranging the last term, one gets

\[ n_i \Phi'(V^i) \theta^i \sum_{t} (1 + r)^{2-t} d^i_t + (\gamma^i - \gamma^{i+1}) \theta^i \sum_{t} (1 + r)^{2-t} d^i_t - \lambda n_i \sum_{t} (1 + r)^{2-t} d^i_t \]

\[ + \lambda n_i (\alpha - 1) \sum_{t} \left( q_c \frac{\partial d^i_t}{\partial \gamma^i} \sum_{t} (1 + r)^{2-t} d^i_t \right) (1 + r)^{2-t} = 0 \quad \forall i. \]

Substituting the Slutsky equations (29) and (30) into this and summing over all \( i \) gives

\[ n_i \sum \Phi'(V^i) \theta^i \sum_{t} (1 + r)^{2-t} d^i_t + \sum_{i} (\gamma^i - \gamma^{i+1}) \theta^i \sum_{t} (1 + r)^{2-t} d^i_t - \lambda \sum_i n_i \sum_{t} (1 + r)^{2-t} d^i_t \]

\[ + \lambda \sum_{i} n_i (\alpha - 1) \sum_{t} \left( \frac{\partial \tilde{d}^i_t}{\partial \gamma^i} - \frac{\partial d^i_t}{\partial \alpha} \right) (1 + r)^{2-t} = 0, \]

which, after substituting for (24) yields

\[ \lambda \sum_{i} n_i (\alpha - 1) \sum_{t} \frac{\partial \tilde{d}^i_t}{\partial \alpha} (1 + r)^{2-t} = 0. \]

This requires that \( \alpha^* = 0. \)
Proposition 2: If in the optimum \( \mu^i = 0, \forall i \), so policy relaxes all credit constraints in the economy, then \( t^*_c = t^*_d < 0 \).

Proof: Multiply (22) by \( \sum_t c^i_t (1 + r^i_t)^{2-t} \), then substitute for compensated demands, and substitute the result it into (24). Using \( r_i = r, \forall i \), obtain

\[
\lambda t_c \sum_i n_i \sum_t \frac{\partial c^i_t}{\partial q_c} (1 + r_i) (1 + r_g)^{2-t} + \lambda t_d \sum_i n_i \sum_t \frac{\partial d^i_t}{\partial q_c} (1 + r_i) (1 + r_g)^{2-t} + \lambda \sum_i n_i (r_g - r) c^i_1 = 0.
\]  

(35)

Multiply (22) by \( \sum_t c^i_t (1 + r^i_t)^{2-t} \), then use compensated demands and substitute the result into (25). Obtain

\[
\lambda t_c \sum_i n_i \sum_t \frac{\partial c^i_t}{\partial q_d} (1 + r_g)^{2-t} + \lambda t_d \sum_i n_i \sum_t \frac{\partial d^i_t}{\partial q_d} (1 + r_g)^{2-t} + \lambda \sum_i n_i (r_g - r) d^i_1 = 0.
\]  

(36)

Equations (35) and (36) characterize \((t^*_c, t^*_d)\) when optimal income taxes are optimal and all individuals are unconstrained. In matrices, the system can be expressed as

\[
\begin{pmatrix}
\sum_i n_i \sum_t \frac{\partial c^i_t}{\partial q_c} (1 + r_i) (1 + r_g)^{2-t} \\
\sum_i n_i \sum_t \frac{\partial c^i_t}{\partial q_d} (1 + r_g)^{2-t}
\end{pmatrix}
\begin{pmatrix}
t_c \\
t_d
\end{pmatrix}
=
\begin{pmatrix}
-(r_g - r) \sum_i n_i c^i_1 / \lambda \\
-(r_g - r) \sum_i n_i d^i_1 / \lambda
\end{pmatrix}
\]

where the leftmost matrix is \((2 + r_g)\) times the per-period Slutsky matrix. With time-separable utility, this matrix is necessarily negative semi-definite. We denote it by \( S \) and its determinant by \( det(S) > 0 \). By Cramer’s rule

\[
t_c = \frac{r_g - r}{\lambda det(S)} \times \left( \sum_i n_i \sum_t \frac{\partial d^i_t}{\partial q_d} (1 + r_g)^{2-t} \sum_i n_i c^i_1 - \sum_i n_i \sum_t \frac{\partial d^i_t}{\partial q_c} (1 + r_g)^{2-t} \sum_i n_i d^i_1 \right) < 0
\]  

(37)
\[ t_d = \frac{r_g - r}{\lambda \det(S)} \times \left( \sum_i n_i \sum_t \frac{\partial \tilde{c}_i}{\partial q_c} (1 + r_g)^{2-t} \sum_i n_i d_i^t - \sum_i n_i \sum_t \frac{\partial \tilde{d}_i}{\partial q_c} (1 + r_g)^{2-t} \sum_i n_i c_i^t \right) < 0 \]

(38)

The properties of the Slutsky matrix directly imply that (37) and (38) are negative since goods \( c \) and \( d \) are net substitutes.

To prove differentiation differentiation, let us convert per-unit taxes into ad-valorem tax rates \((\tau_c, \tau_d) \equiv (\frac{t_c}{q_c}, \frac{t_d}{q_d})\). Thus, we are interested in the sign of \( \tau_c - \tau_d \equiv \frac{t_c}{q_c} - \frac{t_d}{q_d} \), or in the difference between \( t_c q_d \) and \( t_d q_c \). Use the identity \( r_g \equiv r_g \pm r \), the homogeneity of the Slutsky matrix, and divide all terms by \( r_g - r \), which result tell us that differentiation occurs if

\[
\sum_i c_i^t \sum_i n_i q_d \frac{\partial \tilde{d}_i}{\partial q_d} - \sum_i d_i^t \sum_i n_i q_c \frac{\partial \tilde{c}_i}{\partial q_c} \neq \sum_i c_i^t \sum_i n_i q_c \frac{\partial \tilde{c}_i}{\partial q_c} - \sum_i d_i^t \sum_i n_i q_d \frac{\partial \tilde{d}_i}{\partial q_d}.
\]

Cancelling similar terms and regrouping terms of the same sign together, differentiation occurs if

\[
\sum_i c_i^t \sum_i n_i q_d \frac{\partial \tilde{d}_i}{\partial q_d} + \sum_i c_i^t \sum_i n_i \frac{q_c}{q_c} \frac{\partial \tilde{c}_i}{\partial q_c} \neq \sum_i d_i^t \sum_i n_i \frac{q_c}{q_c} \frac{\partial \tilde{d}_i}{\partial q_d} + \sum_i d_i^t \sum_i n_i \frac{q_d}{q_d} \frac{\partial \tilde{d}_i}{\partial q_c}
\]

where both sides equal zero, again, by the per-period homogeneity properties of Slutsky sub-matrices under time-separability. Therefore, the proof is completed and taxes are undifferentiated and negative.
**Proposition 3:** Let us denote $B_i \equiv (\mu_i/\lambda)(\Phi'(V_i) + \gamma^i/n_i - \gamma^{i+1}/n_i) - (r_g - r_i)$. Also denote by $C \neq \emptyset$ the subset of types whose credit constraint bind in the optimum: $i \in C \Leftrightarrow \mu_i > 0$. Then, the optimal policy has $t_d > t_c$ if and only if

$$\left( \sum_{i \in C} n_i B_i c_i^1 \right) - \left( \sum_{i \in C} n_i B_i d_i^1 \right) > 0.$$

**Proof:** Let us characterize the optimal commodity tax system when nonlinear income taxes are optimal. Let us suppose that, in the optimal tax system, $i \in C$ if the individual is constrained and $i \in U$ if he is unconstrained, with $C \neq \emptyset$. To characterize $q^*_c$, multiply (22) by $\sum_t (1 + r_g)^{2-t} c_t^i$, substitute compensated demands into it, aggregate over all $i$, and substitute the result into (24). We obtain that $q^*_c$ is characterized by:

$$\lambda \sum_{i \in U} n_i \left[ t_c \sum_t (1 + r_g)^{2-t} \frac{\partial \tilde{c}_i^1}{\partial q_c} + \lambda t_d \sum_t (1 + r_g)^{2-t} \frac{\partial \tilde{d}_i^1}{\partial q_c} \right]$$

$$+ \lambda \sum_{i \in C} n_i \left[ t_c \left( \frac{\partial c_i^1}{\partial q_c} (1 + r_g) + \frac{\partial \tilde{c}_i^1}{\partial q_c} + \frac{\partial c_i^2}{\partial I} c_i^1 (1 + r_i) \right) + t_d \left( \frac{\partial d_i^1}{\partial q_c} (1 + r_g) + \frac{\partial \tilde{d}_i^1}{\partial q_c} + \frac{\partial c_i^2}{\partial I} c_i^1 (1 + r_i) \right) \right]$$

$$+ \lambda \sum_i n_i (r_g - r_i)c_i^1 - \sum_i (n_i \Phi'(V_i) + \gamma_i - \gamma^{i+1})c_i^1 = 0$$

Using compensated demands to simplify terms for $i \in C$, we use

$$\frac{\partial c_i^1}{\partial q_c} (1 + r_g) + \frac{\partial \tilde{c}_i^1}{\partial q_c} + \frac{\partial c_i^2}{\partial I} c_i^1 (1 + r_i) = \frac{\partial c_i^1}{\partial q_c} (1 + r_g) - c_1 \frac{\partial c_i^1}{\partial \phi} (1 + r_g) + \frac{\partial \tilde{c}_i^1}{\partial q_c} + \frac{\partial c_i^2}{\partial I} c_i^1 (1 + r_i)$$

$$= \sum_i (1 + r_g)^{2-t} \frac{\partial \tilde{c}_i^1}{\partial q_c} + \left( \frac{\partial c_i^2}{\partial I} - \frac{\partial \tilde{c}_i^1}{\partial \phi} \right) c_i^1 (1 + r_i) - (r_g - r_i)c_i^1 \frac{\partial c_i^1}{\partial \phi}.$$
condition with respect to \( q_c \),

\[
\lambda t_c \sum_i n_i \sum_t \frac{\partial c_i}{\partial q_c} (1 + r_g)^{2-t} + \lambda t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_i}{\partial q_c} (1 + r_g)^{2-t} \\
+ \lambda r_g \sum_{i \in \mathcal{C}} n_i c_i^1 \frac{\partial c_i}{\partial \phi} - \lambda r_g \sum_{i \in \mathcal{C}} n_i c_i^1 \frac{\partial d_i}{\partial \phi}
\]

\[
+ \lambda \sum_i n_i (r_g - r_i) c_i^1 - \sum_i \mu \left[ n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1} \right] c_i^1 = 0.
\] (39)

Then making use of the linearity of Engel’s curves,

\[
\lambda t_c \sum_i n_i \sum_t \frac{\partial c_i}{\partial q_c} (1 + r_g)^{2-t} + \lambda t_d \sum_i n_i \sum_t \frac{\partial \tilde{d}_i}{\partial q_c} (1 + r_g)^{2-t} \\
+ \lambda r_g \sum_{i \in \mathcal{C}} n_i c_i^1 \frac{\partial c_i}{\partial \phi} - \lambda r_g \sum_{i \in \mathcal{C}} n_i c_i^1 \frac{\partial d_i}{\partial \phi}
\]

\[
+ \lambda \sum_i n_i (r_g - r_i) c_i^1 - \sum_i \mu \left[ n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1} \right] c_i^1 = 0.
\] (40)

Performing the same operations on the good \( d \) and expressing the system in matrices, get

\[
\begin{pmatrix}
  s_{cc} - r_g \sum_{i \in \mathcal{C}} n_i c_i^1 \frac{\partial c_i}{\partial \phi} & s_{cd} - r_g \sum_{i \in \mathcal{C}} n_i c_i^1 \frac{\partial d_i}{\partial \phi} \\
  s_{dc} - r_g \sum_{i \in \mathcal{C}} n_i d_i^1 \frac{\partial c_i}{\partial \phi} & s_{dd} - r_g \sum_{i \in \mathcal{C}} n_i d_i^1 \frac{\partial d_i}{\partial \phi}
\end{pmatrix}
\begin{pmatrix}
  t_c \\
  t_d
\end{pmatrix}
\]

\[
= \begin{pmatrix}
  \sum_i \mu \left[ n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1} \right] c_i^1 / \lambda - \sum_i n_i (r_g - r_i) c_i^1 \\
  \sum_i \mu \left[ n_i \Phi'(V^i) + \gamma^i - \gamma^{i+1} \right] d_i^1 / \lambda - \sum_i n_i (r_g - r_i) d_i^1
\end{pmatrix}
\]

Call the lefmost, \( 2 \times 2 \) matrix of the system, \( S' \). Its determinant is unambiguously positive:

\[
\det(S') = [s_{cc} s_{dd} - s_{cd}^2]
\]
\[-r_g s_{cc} \sum_{i \in C} n_i d^i_1 \frac{\partial d^i_1}{\partial \phi} - r_g s_{dd} \sum_{i \in C} n_i c^i_1 \frac{\partial c^i_1}{\partial \phi} + r_g s_{cd} \sum_{i \in C} n_i d^i_1 \frac{\partial c^i_1}{\partial \phi} + r_g s_{cd} \sum_{i \in C} n_i c^i_1 \frac{\partial d^i_1}{\partial \phi} \]

where the first line above is positive (since \(S\) is negative semi-definite and of size 2 \(\times\) 2) and the second line is also positive, since \(c\) and \(d\) are normal and net substitutes.

**Optimal commodity taxes**

Using Cramer’s rule,

\[
t^*_c = \frac{1}{\det(S')} \left[ \sum_i n_i B_i c^i_1 \left( s_{dd} - r_g \sum_{i \in C} n_i d^i_1 \frac{\partial d^i_1}{\partial \phi} \right) - \sum_i n_i B_i d^i_1 \left( s_{cd} - r_g \sum_{i \in C} n_i c^i_1 \frac{\partial d^i_1}{\partial \phi} \right) \right]
\] (41)

and

\[
t^*_d = \frac{1}{\det(S')} \left[ \sum_i n_i B_i d^i_1 \left( s_{cc} - r_g \sum_{i \in C} n_i c^i_1 \frac{\partial c^i_1}{\partial \phi} \right) - \sum_i n_i B_i c^i_1 \left( s_{cd} - r_g \sum_{i \in C} n_i c^i_1 \frac{\partial c^i_1}{\partial \phi} \right) \right]
\] (42)

**Differentiation**

To extract some intuition out of these formulas, we first want to find a way to use the homogeneity of the Slutsky Matrices. Make use of ad valorem taxes \(\tau_c = t_c/q_c\) and \(\tau_d = t_d/q_d\) :

\[
\tau^*_c = \frac{1}{q_c \times \det(S')} \left[ \sum_i n_i B_i c^i_1 \left( s_{dd} - r_g \sum_{i \in C} n_i d^i_1 \frac{\partial d^i_1}{\partial \phi} \right) - \sum_i n_i B_i d^i_1 \left( s_{cd} - r_g \sum_{i \in C} n_i c^i_1 \frac{\partial d^i_1}{\partial \phi} \right) \right]
\]

and

\[
\tau^*_d = \frac{1}{q_d \times \det(S')} \left[ \sum_i n_i B_i d^i_1 \left( s_{cc} - r_g \sum_{i \in C} n_i c^i_1 \frac{\partial c^i_1}{\partial \phi} \right) - \sum_i n_i B_i c^i_1 \left( s_{cd} - r_g \sum_{i \in C} n_i c^i_1 \frac{\partial c^i_1}{\partial \phi} \right) \right].
\]

We know that \(\tau_d > \tau_c\) iff \(t_d q_c - t_c q_d > 0\). Making use of the two equations above, obtain the
condition:

\[ \text{det}(S')(t_d q_c - t_c q_d) = \]

\[ \left[ \sum_i n_i B_i d_i^r \left( q_c s_{cc} - r_g q_c \sum_{i \in C} n_i c_i \frac{\partial c_i}{\partial \phi} \right) - \sum_i n_i B_i c_i \left( q_c s_{cd} - r_g q_c \sum_{i \in C} n_i d_i \frac{\partial c_i}{\partial \phi} \right) \right] \]

\[ - \left[ \sum_i n_i B_i c_i \left( q_d s_{dd} - r_g q_d \sum_{i \in C} n_i d_i \frac{\partial d_i}{\partial \phi} \right) - \sum_i n_i B_i d_i \left( q_d s_{cd} - r_g q_d \sum_{i \in C} n_i c_i \frac{\partial d_i}{\partial \phi} \right) \right]. \]

Using the properties of Slutsky matrices, which implies that \( \sum_i n_i B_i d_i^r [q_c s_{cc} + q_d s_{cd}] = 0; \sum_i n_i B_i c_i [q_d s_{dd} + q_c s_{cd}] = 0 \), the expression reduces to

\[ \text{det}(S')(t_d q_c - t_c q_d) = \]

\[- \sum_i n_i B_i d_i^r \sum_{i \in C} n_i c_i \frac{\partial c_i}{\partial \phi} + \sum_i n_i B_i c_i \sum_{i \in C} n_i d_i^r \frac{\partial d_i}{\partial \phi} \]

Walras' law tells us that \( q_c \frac{\partial c_i}{\partial \phi} + q_d \frac{\partial d_i}{\partial \phi} = 1 \) for constrained individuals. Linear Engel's curves tell us that \( \frac{\partial c_i}{\partial \phi} = \frac{\partial c_1}{\partial \phi}, \forall i \in C \) and the same for good \( d \). Therefore, re-expressing again,

\[ \text{det}(S')(t_d q_c - t_c q_d) = \sum_i n_i B_i c_i \sum_{i \in C} n_i d_i^r - \sum_i n_i B_i d_i \sum_{i \in C} n_i c_i. \]

Therefore, \( t_d > t_c \) if and only if

\[ \left( \sum_i n_i B_i c_i \frac{\partial c_i}{\partial \phi} \sum_{i \in C} n_i d_i^r - \sum_i n_i B_i d_i \frac{\partial d_i}{\partial \phi} \right) > 0. \]
Figure 1: Optimal Labor Income Tax Rates under Different Scenarios

C Tables and Figures
Table 1: Characteristics of Optimal Allocation under Different Scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$q_c$</th>
<th>$q_d$</th>
<th>$q_d/q_c$</th>
<th>% Constrained</th>
<th>$\sum_i n_iV_i$</th>
<th>$%\Delta^L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Credit Constraint</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0%</td>
<td>16.399</td>
<td>10.22%</td>
</tr>
<tr>
<td>Credit Constraint</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>62.23%</td>
<td>16.346</td>
<td>7.01%</td>
</tr>
<tr>
<td>Cred. Const. and Subsidies</td>
<td>0.858</td>
<td>0.881</td>
<td>1.028</td>
<td>42.96%</td>
<td>16.35</td>
<td>7.28%</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations. Numbers rounded.

Table 2: Welfare gains of subsidies as a percentage of the No credit constraint scenario

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>No subsidies</th>
<th>With subsidies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>60.8%</td>
<td>72.7%</td>
</tr>
<tr>
<td>0.5</td>
<td>29.6%</td>
<td>83.6%</td>
</tr>
<tr>
<td>0.7</td>
<td>8.8%</td>
<td>94.6%</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations. Numbers rounded.

Figure 2: Prices and Optimal Commodity Taxes under Different Interest Rate Spreads
Figure 3: Optimal Labor Income Tax Rates under Different Interest Rate Spreads
Figure 4: Prices and Optimal Commodity Taxes under Different levels of Basic Need
Figure 5: Optimal Labor Income Tax Rates under Different levels of Basic Need